Graphical Rasch models

Rasch models embedded in a multivariate graphical model

Definition of IRT and Rasch graphs

Tests of conditional independence

Parametrizing Rasch models

Testing fit of items to Rasch models

Estimating person parameters and assessment of measurement quality

Demo

What to do when fit is rejected

The CONF08 example

The data for this project originated in the Danish component of the European values studies in 2008. We use five polytomous items measuring confidence in the following public institutions.

D) The police

E) The parliament

G) The social security system

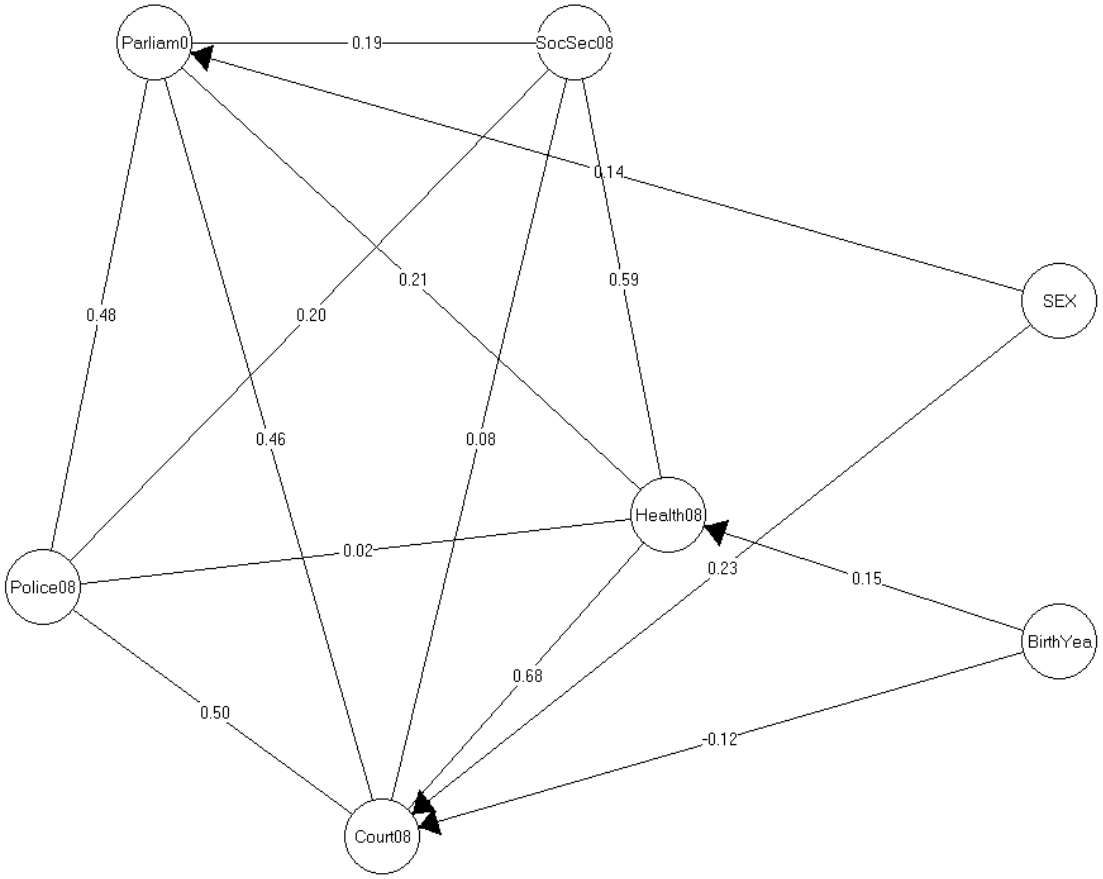
K) The health system

L) The courts.

Response categories were "High degree of confidence" (0), "Some degree of confidence" (1), "Little degree of confidence" (2) and "No confidence" (3). Responses were coded in such a way that a high item score indicate a high degree of distrust. We refer to the instrument as the CONF scale even though a high CONF score is an indication of lack of confidence of public institutions.

In addition to these variables, we have included information on sex (Y) and age (Z) to illustrate graphical Rasch models with exogenous items.

The *graphical* model defined by screening



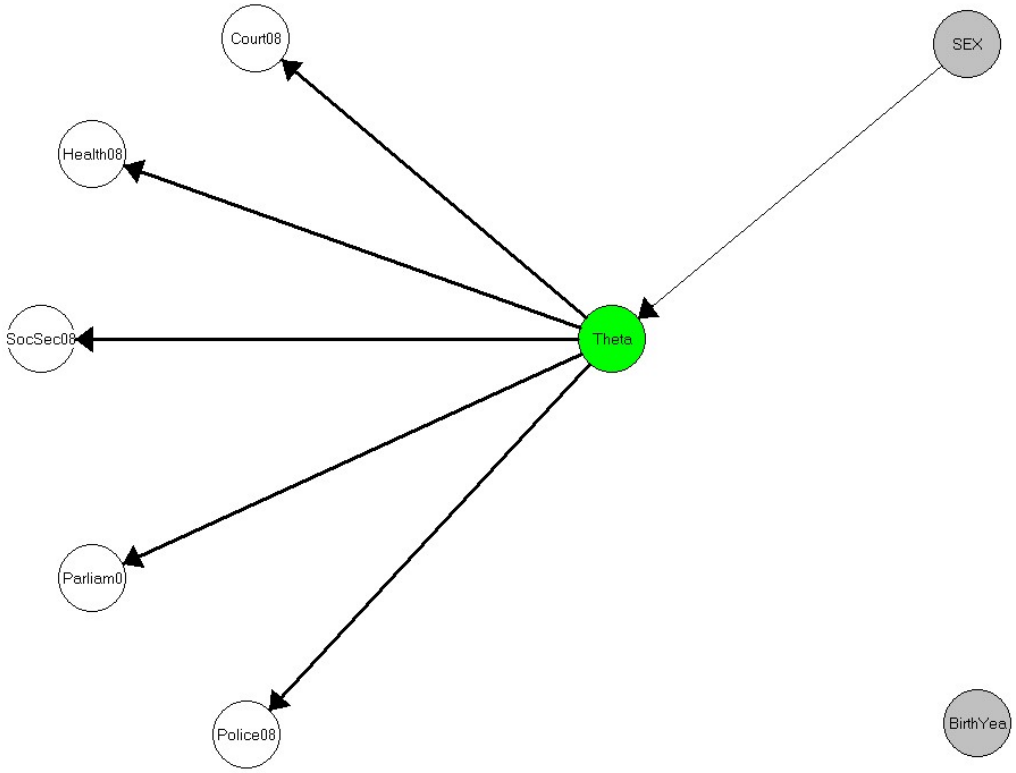
Can we simplify things by replacing the five questions concerning public institutions with a summery confidence score?

Yes, if items fit a Rasch model

The IRT graph

IRT and Rasch models assume that items depend on an unobserved latent variable T that explains the correlation among items and among items and exogenous variables.

Adding the latent variable to the model defines an extended graphical model.



The graphical Rasch model

$$\Pr(D,E,G,K,L \mid T=\theta,Y,Z) = \Pr(D \mid T=\theta)\Pr(E \mid T=\theta)\Pr(G \mid T=\theta)\Pr(K \mid T=\theta)\Pr(L \mid T=\theta)$$

The outcome of the latent variable (θ) appear as an unknown parameter in the conditional distribution of the items given θ

The IRT model is a Rasch model

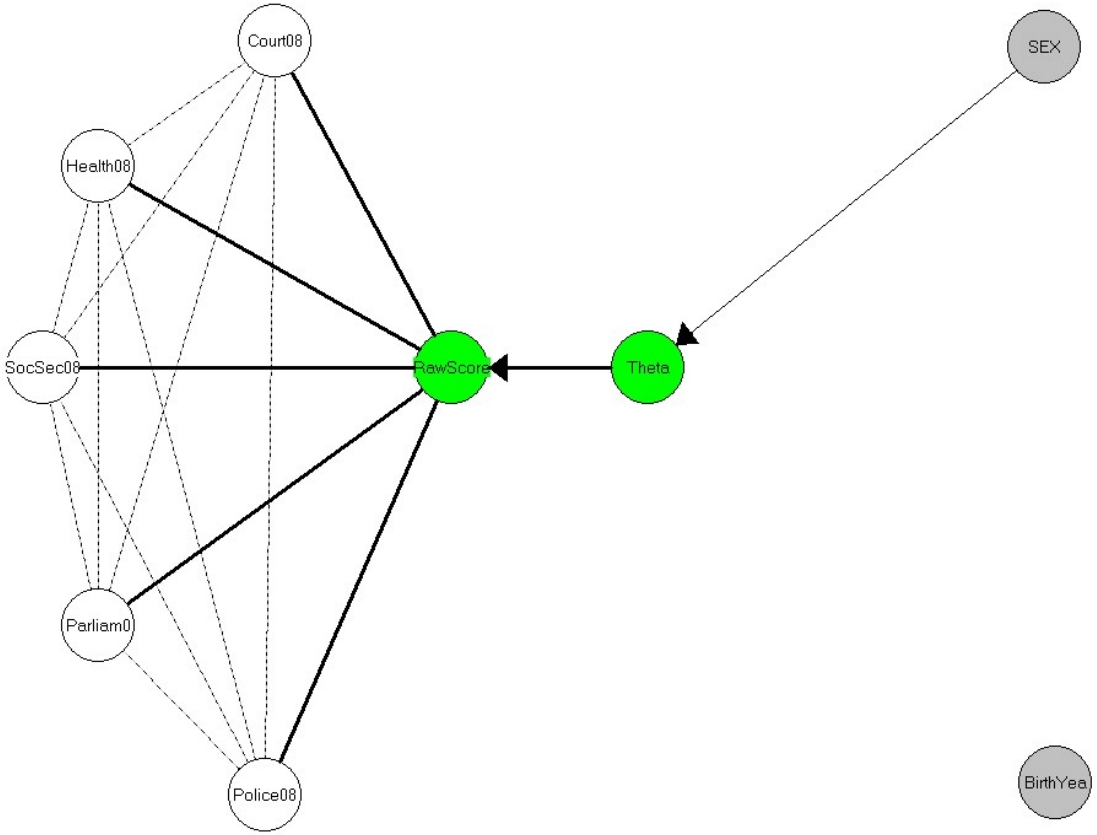
if and only if

the score over all items is sufficient for θ

This implies that items are conditionally independent of T given the score

Adding the score to the model generates a new graphical model.

The Rasch graph



This model implies that every item is conditionally independent of sex and age given the raw score (no DIF)

Item ⊥ Sex | Score and Item ⊥ Age | Score

Tests of no differential item functioning in DIGRAM

```
Analysis of DIF relative to Y: SEX
Scale : # - RawScore

     Item    X²    df asymp exact  gamma asymp exact nsim
-------------------------------------------------------
D:Police08  28.2 19 0.080 0.071  -0.27 0.003 0.004 1000      --
E:Parliam0  30.4 21 0.084 0.075   0.20 0.023 0.022 1000      +
G:SocSec08  15.3 20 0.760 0.667  -0.06 0.489 0.571   21
K:Health08  19.1 22 0.641 0.524   0.07 0.408 0.333   21
L: Court08  29.8 19 0.055 0.036   0.03 0.723 0.725 1000  *

Analysis of DIF relative to Z: BirthYea
Scale : # - RawScore

     Item    X²    df asymp exact  gamma asymp exact nsim
-------------------------------------------------------
D:Police08  76.4 73 0.370 0.524  -0.04 0.542 0.619   21
E:Parliam0 109.5 81 0.019 0.018  -0.11 0.104 0.081 1000  *
G:SocSec08  87.5 78 0.216 0.216   0.11 0.101 0.068 1000
K:Health08 109.5 85 0.038 0.032   0.12 0.058 0.059 1000  *
L: Court08  78.2 74 0.347 0.429  -0.07 0.347 0.286   21
```

Parameterizing Rasch models

Let X be an item. Pr (X=x|θ) can be parametrized in five different ways.

Exponential family **Partial credit version** **Rasch's version**

$$\Pr(X_i = x \mid \theta) = \frac{\exp(x\theta + \sigma_{ix})}{H_i(\theta)} = \frac{\exp(x\theta - \sum_{j=1}^{x} \beta_{ij})}{H_i(\theta)} = \frac{\exp(x(\theta + \sigma_i) + \alpha_{ix})}{H_i(\theta)}$$

Power series versions

$$\Pr(X_i = x \mid \xi = e^{\theta}) = \frac{\xi^x \delta_{ix}}{H_i(\theta)} = \frac{\xi^x \delta_i^x \omega_{ix}}{H_i(\theta)}$$

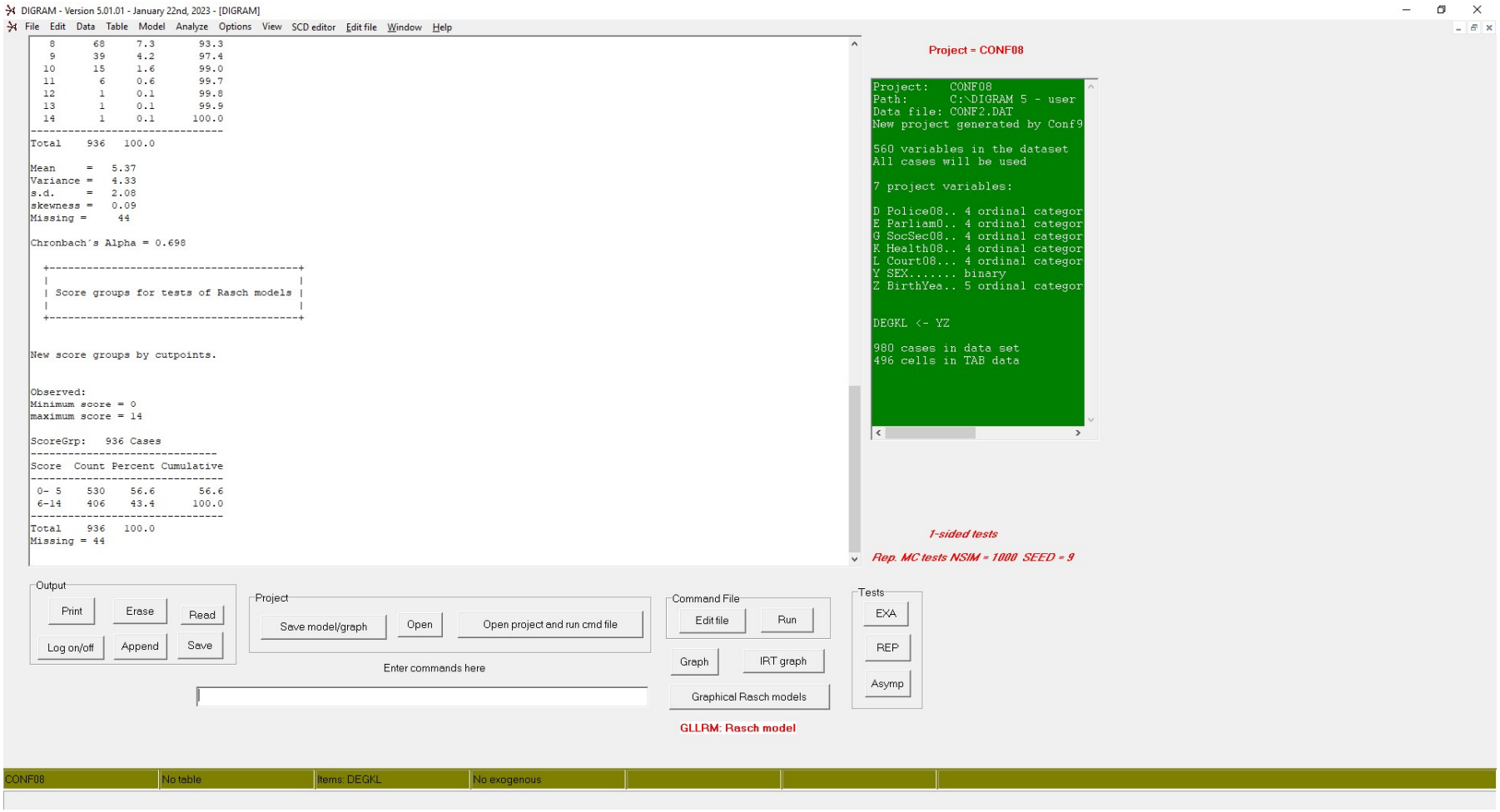
DIGRAM estimates ξ and δ_{ix} and calculates the parameters of the other versions

DIGRAM commands for Rasch analysis

Items DEGKL	**defines the items and calculate to total score**
EXO YZ	**defines the exogenous variables**
DIF	**calculates tests of differential item functioning**
GRM	**invokes the Rasch dialog and calculates the estimates of the item parameters**

After ITEMS DEGKL

The IRT and “Graphical Rasch models” buttons have been enabled and the GLLRM is a Rasch model.



Recall that the test of DIF provided som evidence of DIF

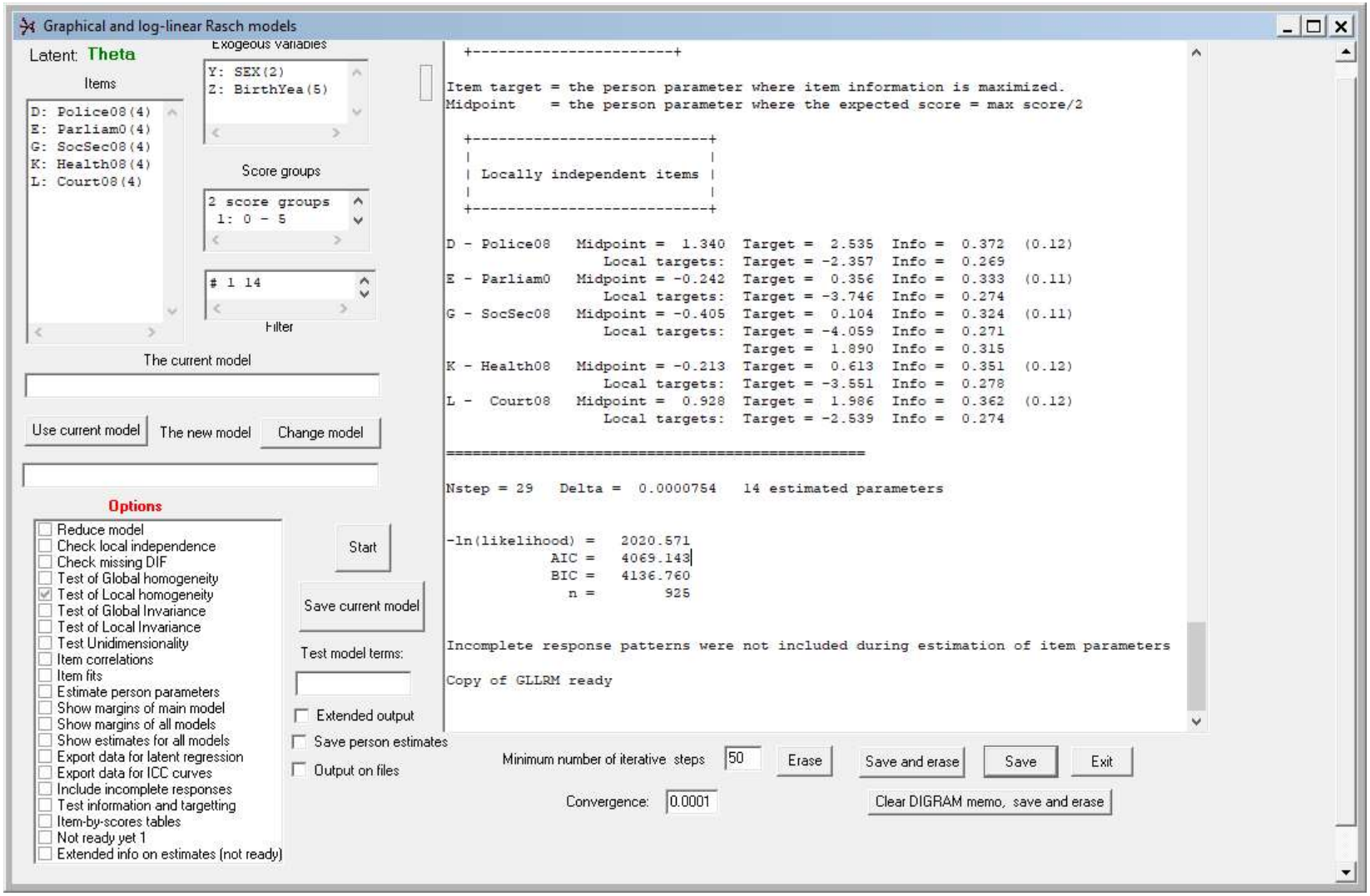
```
Analysis of DIF relative to Y: SEX
Scale : # - RawScore

     Item   X²   df asymp exact  gamma asymp exact nsim
-------------------------------------------------------
D:Police08  28.2 19 0.080 0.071  -0.27 0.003 0.004 1000      --
E:Parliam0  30.4 21 0.084 0.075   0.20 0.023 0.022 1000      +
G:SocSec08  15.3 20 0.760 0.667  -0.06 0.489 0.571   21
K:Health08  19.1 22 0.641 0.524   0.07 0.408 0.333   21
L: Court08  29.8 19 0.055 0.036   0.03 0.723 0.725 1000  *

Analysis of DIF relative to Z: BirthYea
Scale : # - RawScore

     Item   X²   df asymp exact  gamma asymp exact nsim
-------------------------------------------------------
D:Police08  76.4 73 0.370 0.524  -0.04 0.542 0.619   21
E:Parliam0 109.5 81 0.019 0.018  -0.11 0.104 0.081 1000  *
G:SocSec08  87.5 78 0.216 0.216   0.11 0.101 0.068 1000
K:Health08 109.5 85 0.038 0.032   0.12 0.058 0.059 1000  *
L: Court08  78.2 74 0.347 0.429  -0.07 0.347 0.286   21
```

After the GRM command or aclick on “Graphical Rasch models”



Item parameters were estimated. Select options and click START to continue

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| |
| Multiplicative score parameters |
| |
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item	0	1	2	3
D: Police08	1.000	12.680	2.889	0.056
E: Parliam0	1.000	53.375	62.910	4.221
G: SocSec08	1.000	70.883	98.992	7.035
K: Health08	1.000	45.504	51.286	3.969
L: Court08	1.000	15.986	5.635	0.153

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| |
| PCM thresholds and locations |
| |
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item	1	2	3	Location
D: Police08	-2.540	1.479	3.951	0.963
E: Parliam0	-3.977	-0.164	2.702	-0.480
G: SocSec08	-4.261	-0.334	2.644	-0.650
K: Health08	-3.818	-0.120	2.559	-0.459
L: Court08	-2.772	1.043	3.609	0.627

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| |
| Item and category effects |
| |
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item	0	1	2	3	Item effect
D: Police08	0.000	3.503	2.987	-0.000	-0.963
E: Parliam0	0.000	3.497	3.182	-0.000	0.480
G: SocSec08	0.000	3.611	3.294	0.000	0.650
K: Health08	0.000	3.358	3.018	0.000	0.459
L: Court08	0.000	3.398	2.982	0.000	-0.627
---- MICE effects ----					
D: Police08	1.000	33.222	19.835	1.000	0.382
E: Parliam0	1.000	33.027	24.087	1.000	1.616
G: SocSec08	1.000	36.993	26.962	1.000	1.916
K: Health08	1.000	28.741	20.459	1.000	1.583
L: Court08	1.000	29.914	19.730	1.000	0.534

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What do these parameters tell us about the ways that the CONF items function?

Relevant item information

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|                         |
| Midpoints and targets   |
|                         |
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Item target = the person parameter where item information is maximized.
Midpoint    = the person parameter where the expected score = max score/2


D - Police08   Midpoint =  1.340  Target =  2.535  Info =  0.372  (0.12)
                  Local targets:  Target = -2.357  Info =  0.269

E - Parliam0   Midpoint = -0.242  Target =  0.356  Info =  0.333  (0.11)
                  Local targets:  Target = -3.746  Info =  0.274

G - SocSec08   Midpoint = -0.405  Target =  0.104  Info =  0.324  (0.11)
                  Local targets:  Target = -4.059  Info =  0.271
                                  Target =  1.890  Info =  0.315
K - Health08   Midpoint = -0.213  Target =  0.613  Info =  0.351  (0.12)
                  Local targets:  Target = -3.551  Info =  0.278

L -  Court08   Midpoint =  0.928  Target =  1.986  Info =  0.362  (0.12)
                  Local targets:  Target = -2.539  Info =  0.274

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```

Available options for analysis by Rasch models

Test global homogeneity and invariance

Item fits

Check missing DIF

Estimate Person parameters

(Export data for ICC curves)

Test info and targeting

Test global homogeneity and invariance

Erling B. Andersen's conditional likelihood ratio test comparing estimates of item parameters in different score groups and groups defined by outcomes on exogenous variables.

	CLR	df	p
scoregroups	17.6	14	0.227
Y: SEX	36.1	14	0.001
Z: BirthYea	99.9	56	0.000

Items appear to function in the same way for persons with low and high scores, but invariance relative to Sex and age is rejected.

Item fits

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|                                   |
| ** Conditional outfits and infits |
|                                   |
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```

Item	Outfit observed	sd	p	Infit observed	sd	p
D - Police08	0.988	0.058	0.84118	0.991	0.051	0.86451
E - Parliam0	1.012	0.054	0.82591	1.033	0.048	0.49962
G - SocSec08	0.997	0.053	0.95067	1.004	0.048	0.93122
K - Health08	0.995	0.054	0.92024	1.020	0.048	0.67338
L - Court08	0.889	0.057	0.05030	0.906	0.051	0.06568

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|                              |
| Item restscore association   |
|                              |
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Item	Item-restscore gamma observed	expected	sd	p
D - Police08	0.543	0.531	0.035	0.74595
E - Parliam0	0.531	0.539	0.033	0.81189
G - SocSec08	0.528	0.531	0.034	0.91228
K - Health08	0.556	0.541	0.033	0.65789
L - Court08	0.621	0.542	0.034	0.02031

Critical levels adjusted by the Benjamini-Hochberg:

* < 5 % FDR, ** < 1 % FDR, *** = FDR < 0.1 % FDR

Item fit is accepted

Check missing DIF

Conditional likelihood ratio tests that the comparing item parameters of on item in different groups under the assumption that item parameters are the same for the other items

```
Check assumptions of no DIF

Test results will only be shown if p <= 0.05.
Select extended output if you want to see all testresults.

Significant test results:

D & Y:    lr =   17.69  df =   3  p = 0.0005
E & Y:    lr =   10.67  df =   3  p = 0.0137
E & Z:    lr =   27.49  df =  12  p = 0.0066
K & Z:    lr =   21.43  df =  12  p = 0.0444

Benjamini & Hochberg rejects at 0.01500

DIF:              DY EY EZ
```

Estimate Person parameters

ML estimates and WML estimates

Includes estimates of bias and standard errors (of measurement)

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|              |
| ML estimates |
|              |
+--------------+
```

Pseudo ML estimates for extreme scores:
Score = 0 theta = -6.613
Score = 15 theta = 6.202

Properties of ML estimates

Score	Theta estimate	True score	Bias	RMSE	Score SEM
0	-6.613	0.25	0.386	0.824	0.48
1	-5.014	1.00	-0.194	1.112	0.88
2	-3.986	2.00	-0.127	1.077	1.07
3	-3.168	3.00	-0.035	0.967	1.13
4	-2.376	4.00	-0.001	0.900	1.12
5	-1.566	5.00	-0.019	0.867	1.11
6	-0.786	6.00	-0.041	0.853	1.16
7	-0.079	7.00	-0.040	0.835	1.22
8	0.570	8.00	-0.029	0.810	1.26
9	1.184	9.00	-0.016	0.793	1.29
10	1.777	10.00	0.000	0.795	1.30
11	2.365	11.00	0.030	0.827	1.30
12	2.978	12.00	0.084	0.906	1.25
13	3.675	13.00	0.170	1.032	1.13
14	4.633	14.00	0.217	1.079	0.89
15	6.202	14.75	-0.378	0.806	0.49

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|               |
| WML estimates |
|               |
+---------------+
```

Weighted ML estimates

Score	Theta estimate	True score	Bias	RMSE	Score SEM
0	-6.090	0.40	0.519	0.902	0.60
1	-4.732	1.23	0.063	0.942	0.94
2	-3.897	2.10	0.015	0.922	1.08
3	-3.158	3.01	0.015	0.894	1.13
4	-2.393	3.98	0.014	0.890	1.12
5	-1.545	5.03	0.002	0.883	1.11
6	-0.732	6.07	-0.007	0.860	1.16
7	-0.038	7.06	-0.004	0.826	1.22
8	0.597	8.04	-0.001	0.794	1.26
9	1.201	9.03	-0.002	0.773	1.29
10	1.782	10.01	-0.004	0.765	1.30
11	2.348	10.97	-0.005	0.772	1.30
12	2.922	11.91	-0.005	0.802	1.26
13	3.547	12.83	-0.010	0.860	1.16
14	4.322	13.73	-0.063	0.906	0.98
15	5.659	14.59	-0.518	0.892	0.61

Test midpoint = 0.251
Test target = 1.937 Test information = 1.705

The distribution of the person parameter and reliability

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|                                        |
| Monte Carlo estimates of Reliability   |
|                                        |
+----------------------------------------+

Normal trait distribution  mean = -1.33  s.d.=  1.30


Expected True Score =   5.37   Variance =     3.07
Observed mean Score =   5.37   Variance =     4.39

                                 Ratio =     0.70

                Test-retest correlation     =  0.69
                Test-true score correlation =  0.84
```

Test information and targeting

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+--------------------------------+
|                                |
| Targeting and test information |
|                                |
+--------------------------------+
```
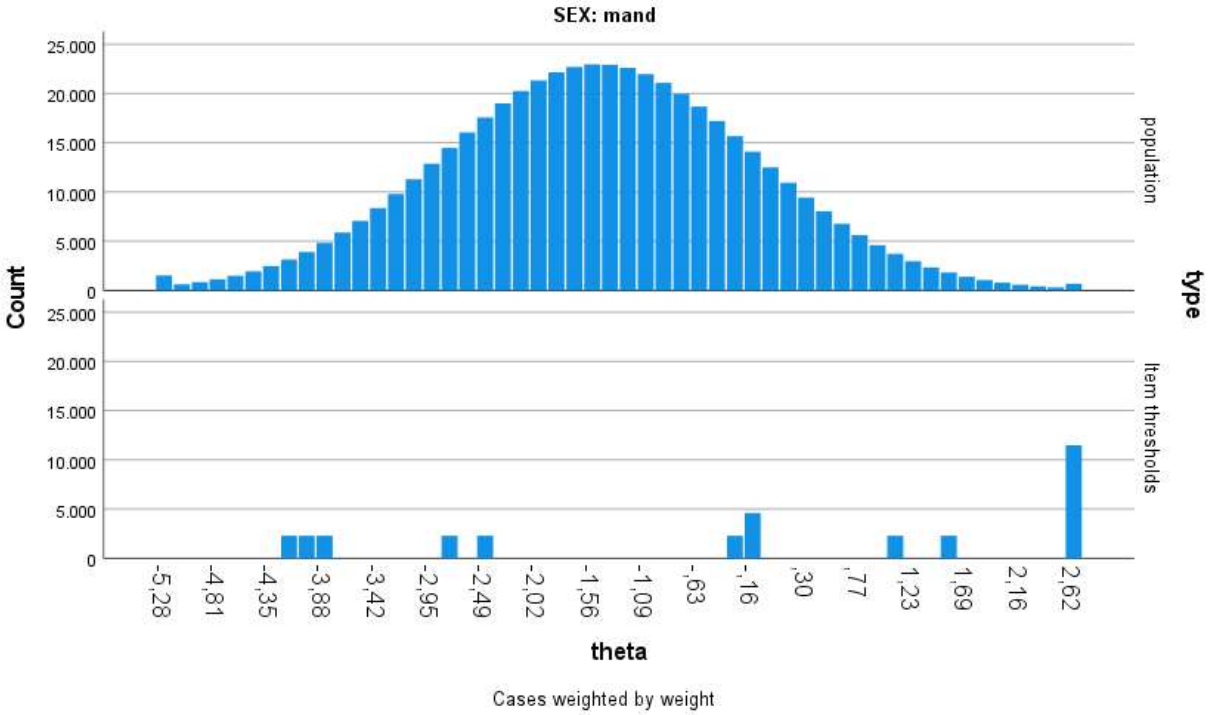
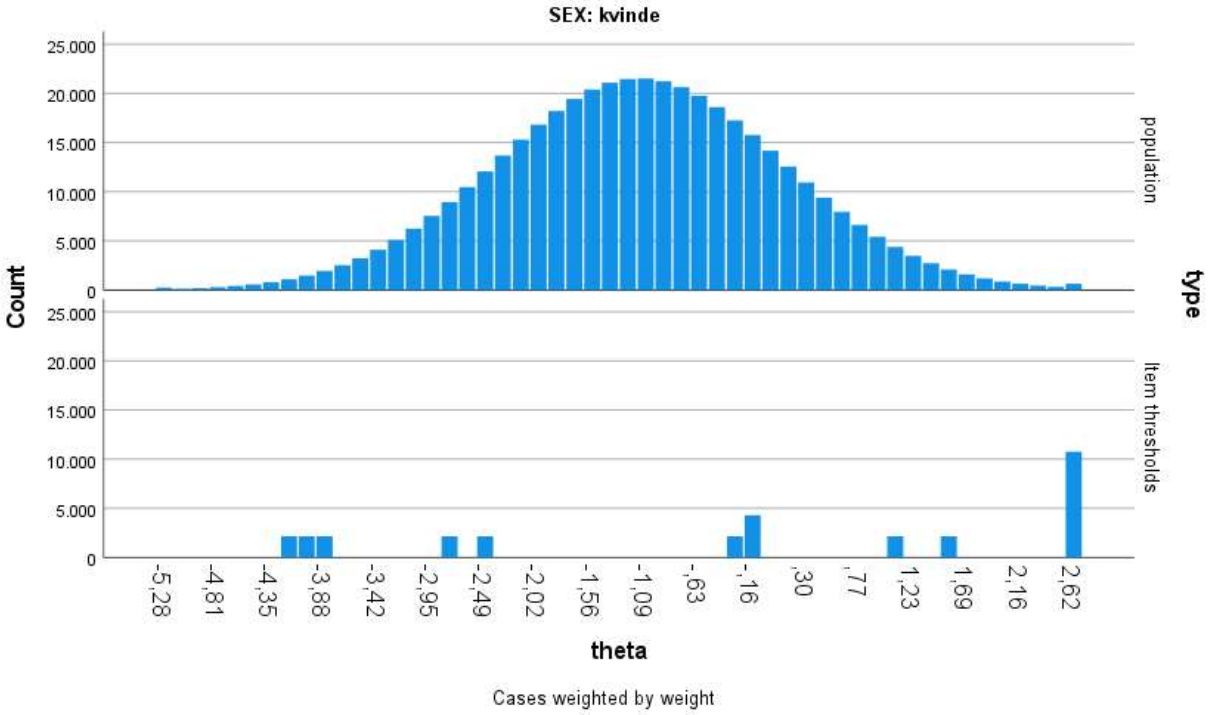

			theta		test info		target	RMSE(WML)		target	
SEX	Target	n	Mean	sd	Mean	max	index	Mean	min	index	PSI
mand	1.94	502	-1.50	1.35	1.317	1.705	0.772	0.869	0.766	0.881	0.728
kvinde	1.94	434	-1.15	1.24	1.348	1.705	0.790	0.861	0.766	0.890	0.697

```
+--------------------------------------+
|                                      |
| True score estimation and reliability |
|                                      |
+--------------------------------------+
```

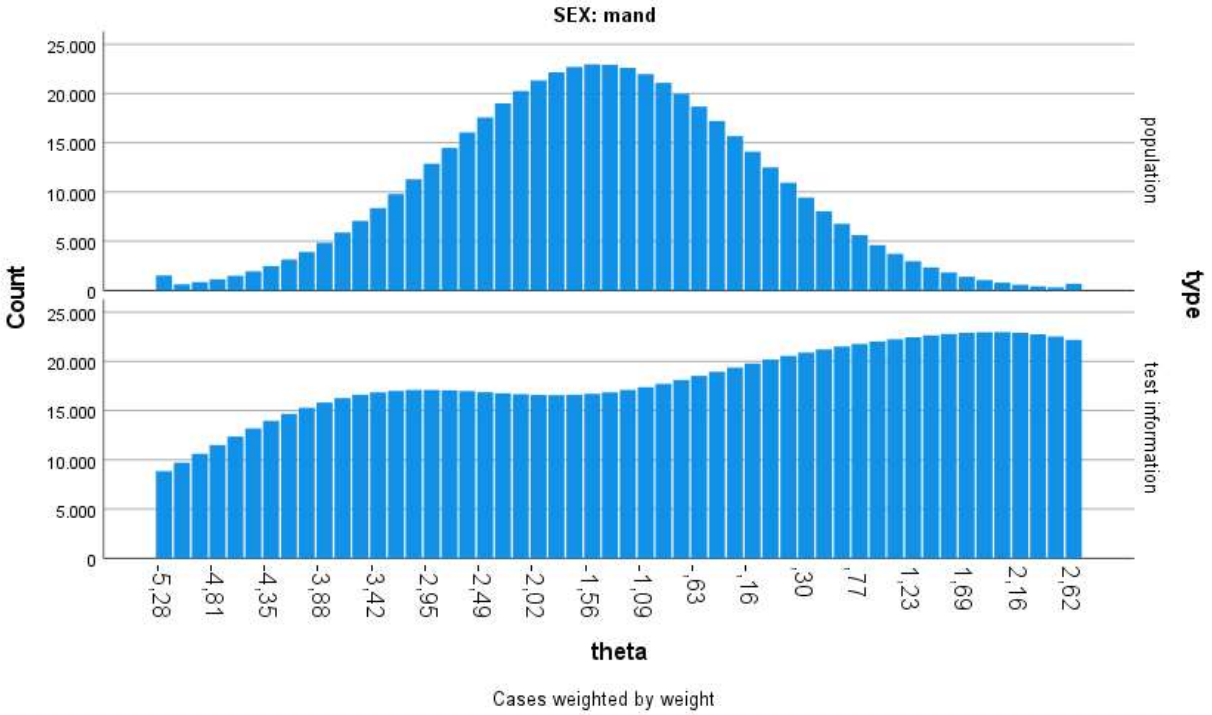
			Score			separation	no sep.	True score SEM	
SEX	Target	n	Mean	sd	reliability	prob	prob	Target	Mean
mand	10.27	502	5.16	2.11	0.693	0.743	0.136	1.31	1.15
kvinde	10.27	434	5.61	2.03	0.664	0.726	0.143	1.31	1.16

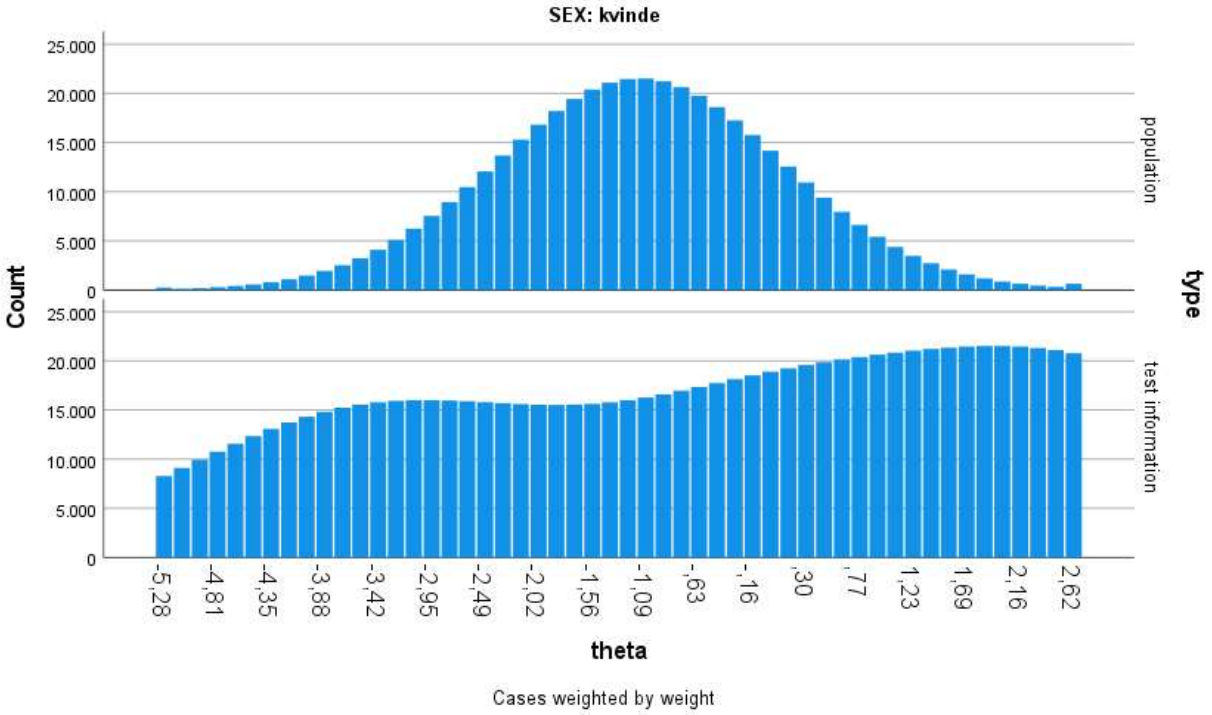
Item Maps


SEX: mand
Count
25.000
20.000
15.000
10.000
5.000
0
population
item thresholds
type
-5,28
-4,81
-4,35
-3,88
-3,42
-2,95
-2,49
-2,02
-1,56
-1,09
-,63
-,16
,30
,77
1,23
1,69
2,16
2,62
theta
Cases weighted by weight

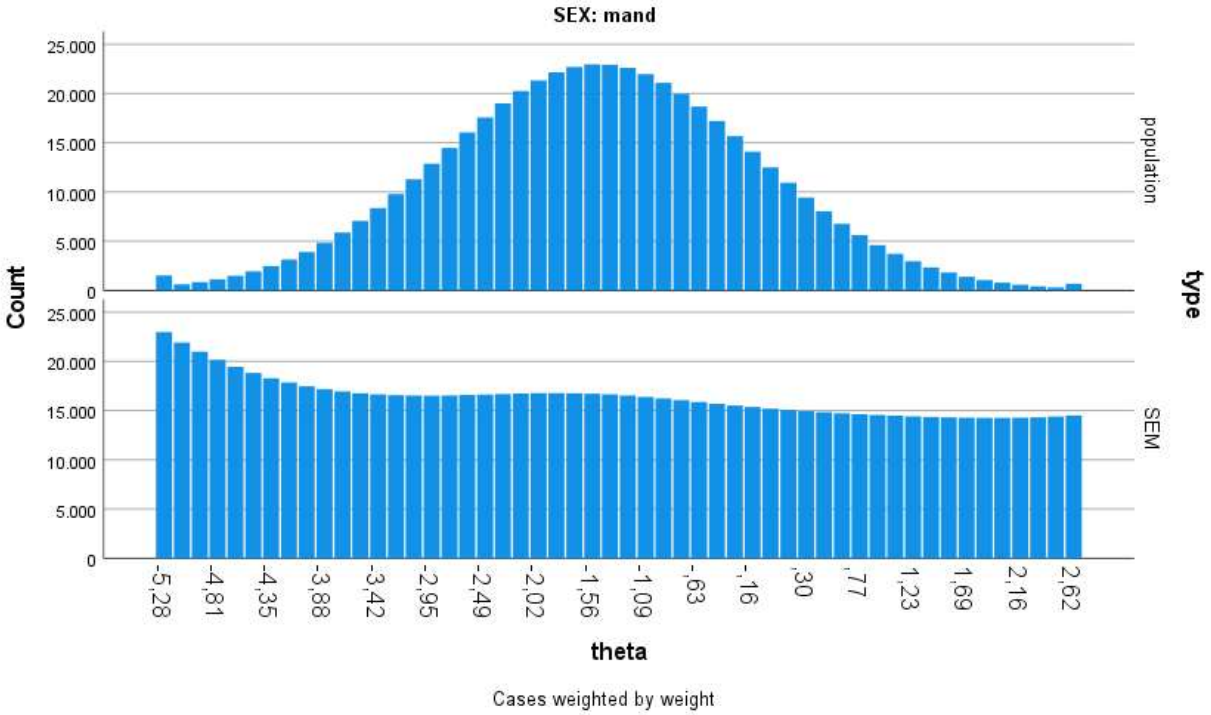

SEX: kvinde
Count
25.000
20.000
15.000
10.000
5.000
0
population
item thresholds
type
-5,28
-4,81
-4,35
-3,88
-3,42
-2,95
-2,49
-2,02
-1,56
-1,09
-,63
-,16
,30
,77
1,23
1,69
2,16
2,62
theta
Cases weighted by weight

Test info maps





SEM maps


SEX: mand
Count
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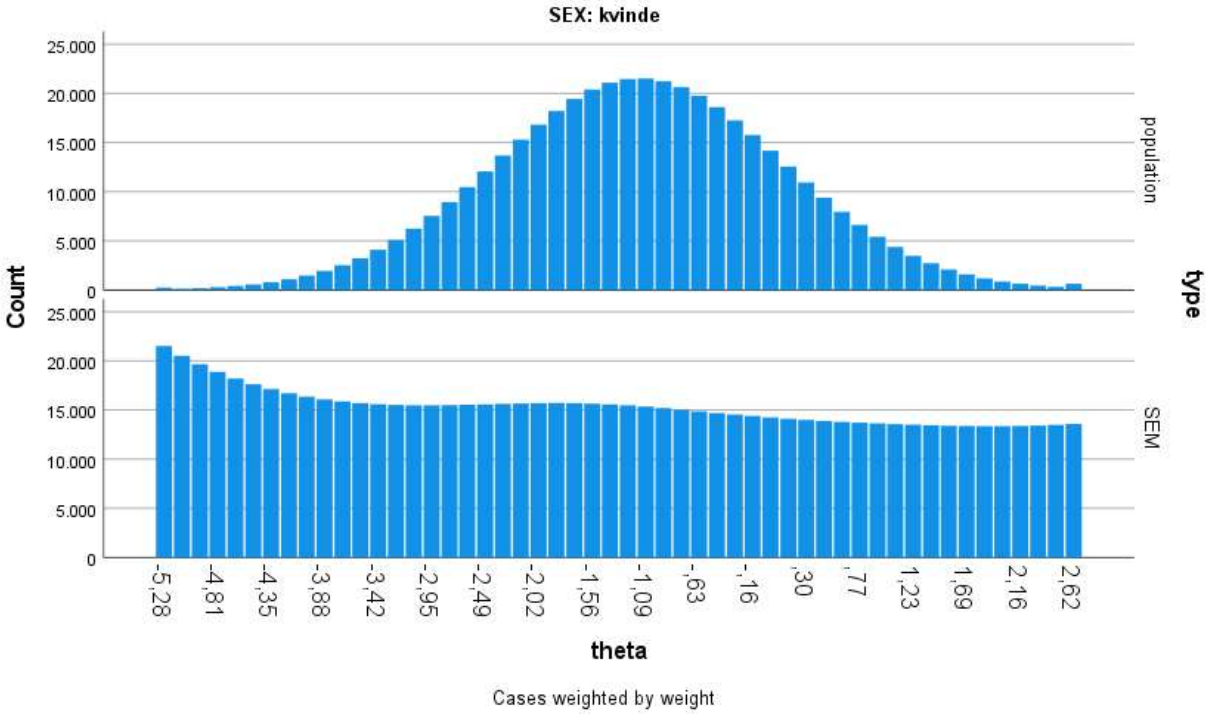
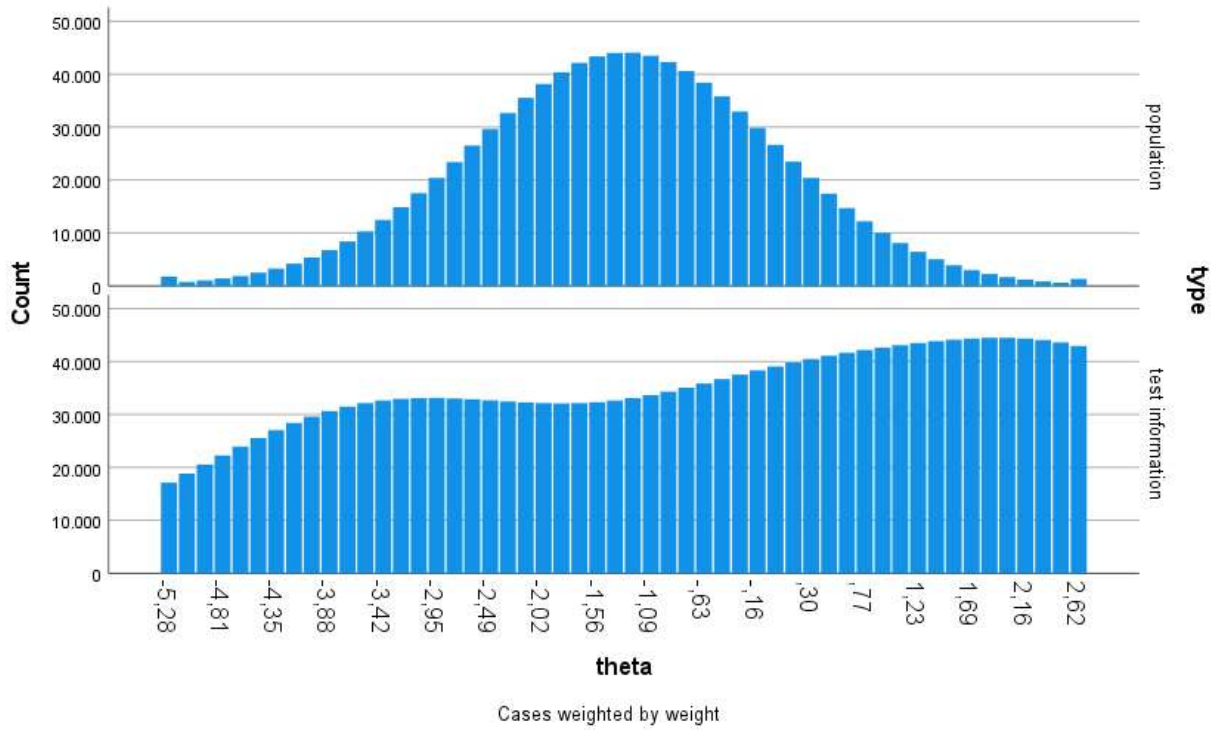
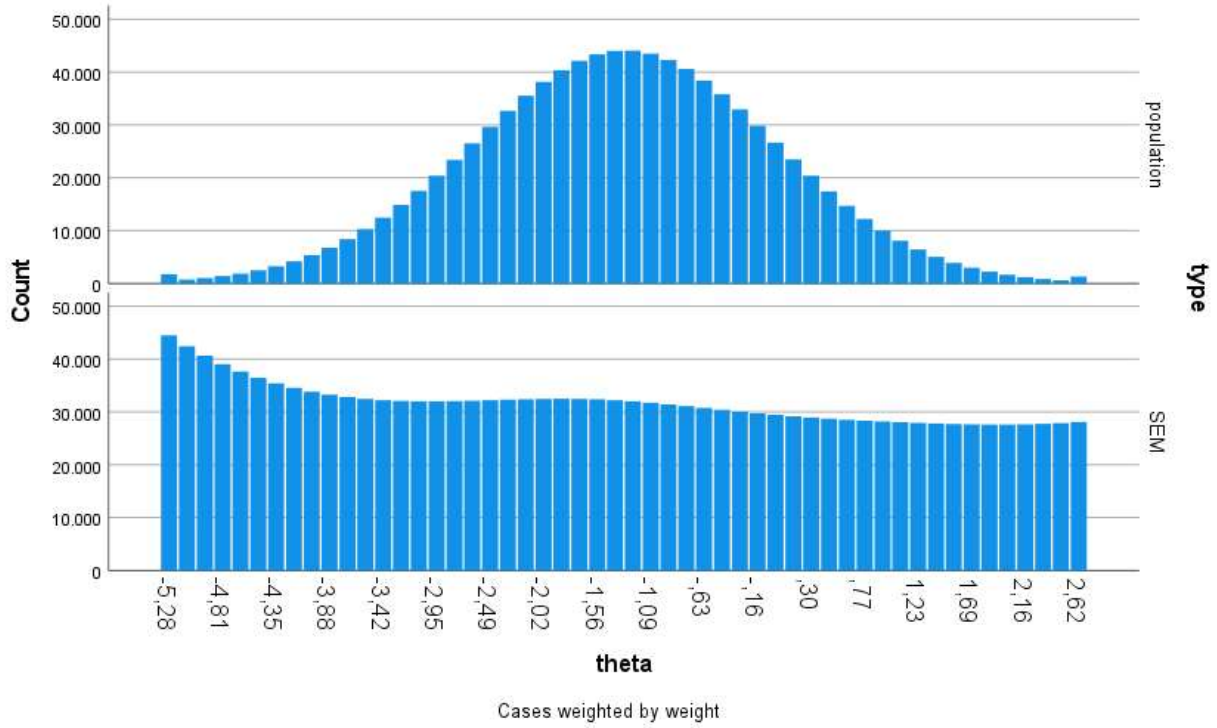
20.000
15.000
10.000
5.000
0
population
SEM
type
-5,28
-4,81
-4,35
-3,88
-3,42
-2,95
-2,49
-2,02
-1,56
-1,09
-,63
-,16
,30
,77
1,23
1,69
2,16
2,62
theta
Cases weighted by weight


SEX: kvinde
Count
25.000
20.000
15.000
10.000
5.000
0
population
SEM
type
-5,28
-4,81
-4,35
-3,88
-3,42
-2,95
-2,49
-2,02
-1,56
-1,09
-,63
-,16
,30
,77
1,23
1,69
2,16
2,62
theta
Cases weighted by weight

Marginal Info & SEM maps

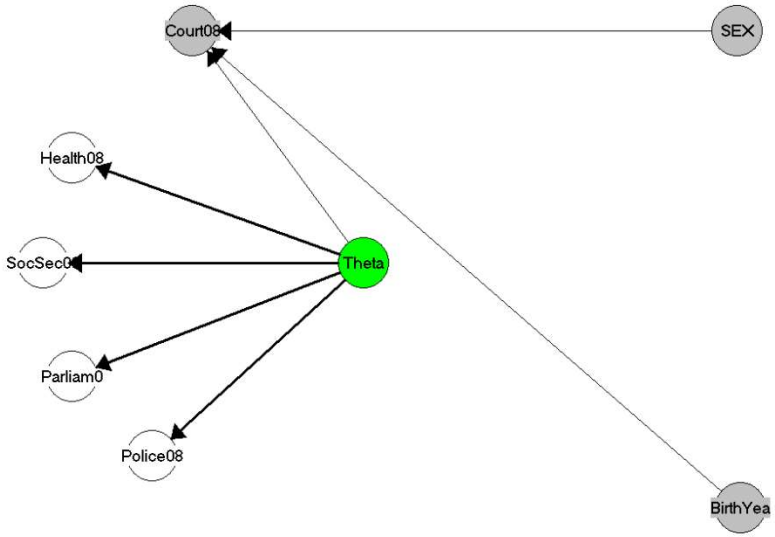




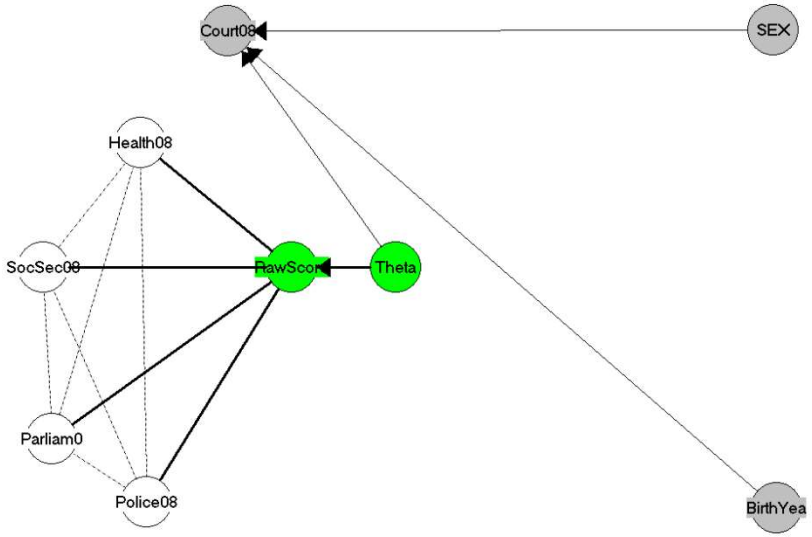
What about the assumption of local independence?

Subtract one item from the model and treat it as an exogenous variable

IRT graph



Rasch graph



If items fit the Rasch models, an item will be conditionally independent of all other items given the rest-score without the item.

Invoking DIGRAM's "SCREEN I" command provide tests of local dependence in Rasch models.

Tests of local independence
- the row item has been subtracted from the score

		D	E	G	K	L
D Police08	Gamma		0.234	-0.124	-0.314	0.163
	p		0.008	0.167	0.001	0.106
E Parliam0	Gamma	0.219		-0.146	-0.313	0.239
	p	0.013		0.088	0.000	0.011
G SocSec08	Gamma	-0.056	-0.203		0.398	-0.222
	p	0.548	0.020		0.000	0.026
K Health08	Gamma	-0.341	-0.278	0.286		0.265
	p	0.000	0.000	0.000		0.002
L Court08	Gamma	0.097	0.101	-0.345	0.147	
	p	0.265	0.258	0.000	0.086	

Even though the CLR test of homogeneity and the tests of item fit accepted the Rasch model, the evidence of DIF and the overwhelming evidence of local dependence show reject the Rasch model.

A graphical and log-linear Rasch model is one option